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Inverse estimation of steady-state surface temperature on a three-dimensional body

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Abstract *This work aims to estimate the unknown surface temperature distribution on the boundary of a heat conducting solid body from temperature measurements taken from another boundary where convective boundary condition is also known. A steady inverse heat conduction problem is described for an arbitrary three-dimensional body. A gradient-based inverse method combined with B-spline function specification is employed to solve the inverse problem. The validity of the proposed method is verified with computational results.*

Nomenclature

Bi = Biot number
D = residual functional of f
f = unknown temperature, °C
$\tilde{f}_k$ = k-th unknown parameters
$\mathbf{f}$ = vector representation of unknown parameters
$\mathbf{g}$ = gradient
h = heat transfer coefficient, $W/m^2\,^\circ C$
H = dimension of slab in z-direction, m
J = residual functional
k = thermal conductivity, W/m°C
L = dimension of slab in x-direction, m
L_2 = space of square-integrable functions
M = number of nodes along axis
n = normal variable on boundary
N = number of base functions
O = order of B-spline functions
P = number of control points along s-direction
Q = number of control points along t-direction
$\mathbf{r}$ = position vector
s = tangent variable on boundary
t = tangent variable on boundary
$\mathbf{S}$ = conjugate direction vector
T_∞ = ambient temperature, °C
v = perturbed temperature
W = dimension of slab in y-direction, m
Y = measured temperature, °C

Greek

β = step length
γ = conjugate coefficients
Δ = variation
φ = B-spline base function

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ϑ = B-spline blending function
λ = adjoint variable
σ = standard deviation of measured temperature

Subscripts
e = exact
i = nodal points in space
k = index to unknown coefficients
p = index to B-spline blending function
q = index to B-spline blending function

Superscripts
p = previous iteration
$+$ = dimensionless variable

Introduction

The inverse heat conduction problem (IHCP) has been widely applied to various thermal systems in order to determine unknowns in heat conduction problems such as boundary conditions, initial conditions, thermophysical properties, and geometry from measurement data. The most extensively investigated IHCP is the determination of the unknown boundary condition from interior temperature measurements (Beck *et al.*, 1985). While transient IHCP has been comprehensively studied, relatively few studies exist regarding steady IHCPs. Of studies that have researched steady IHCP, the following works are noteworthy. Hensel and Hills (1989) solved a two-dimensional steady IHCP using a least squares method incorporated with regularization, which is applicable to an arbitrary geometry. Maillet *et al.* (1991) determined the steady heat transfer coefficients along the surface on a cylinder using an analytical method and boundary element method (BEM). Martin and Dulikravich (1996) also used BEM for two-dimensional steady IHCPs with unknown heat sources and with unknown heat transfer coefficients (Martin and Dulikravich, 1998). Yang *et al.* (1997) estimated surface conditions of a hollow cylinder using a least squares method with a matrix rearrangement technique. Chantasiriwan (2000) proposed an inverse method for evaluating steady-state heat transfer coefficient in a two-dimensional system. Recently, Throne and Olson (2001) compared their singular value decomposition technique with the conventional regularization method for two-dimensional steady IHCP with unknown boundary condition.

In this work, the unknown boundary temperature distribution on a three-dimensional body is estimated from temperature measurements taken from another boundary where convective boundary condition is prescribed. Steady-state, no inside heat generation and linear heat conduction are assumed. The gradient method utilizing the adjoint formulation is selected to solve this steady inverse problem (Alifanov, 1994). The adjoint formulation is comprised of the direct problem, the sensitivity problem and the adjoint problem. BEM is employed for accurate and efficient solution of these three problems. BEM is beneficial for the described IHCP in the following two points. First, in solving the adjoint problem, BEM allows direct evaluation of the boundary flux required for the gradient calculation via solving a linear system of equation, (Brebbia *et al.*, 1984). Second, with BEM, interior nodes are unnecessary for this IHCP since both measurements and estimations are only conducted on the boundary.

Usually, multi-dimensional inverse problems with inside measurements employing many sensors are not very practical. Embedding sensors inside a solid body is not an easy task. Besides, sensor itself can affect the heat conduction. Thus, this study assumes measurement on the boundary only. Thanks to remarkable advances in the thermal imaging technique, accurate temperature distribution on a solid surface can be acquired as a continuous function (Lloyd, 1975). This non-contact, non-destructive thermometry technique has been used widely for remote measurements of the temperatures of both static and moving surfaces, and can perform a lot of tasks that standard sensors such as thermocouples and resistance temperature detectors (RTD) cannot. In heat transfer experiments, the infrared thermographic technology (Ekkad and Han, 1996) and the liquid crystal image method (Mayer *et al.*, 1998) have been employed. Especially, the infrared thermographic technology facilitates simple instrumentation and sensitive measurement (upto 0.05°C). Its detail about instrumentation and application is described by Allison and Gillies (1997). An inverse estimation method utilizing such measurement technique has been already applied to a real-world problem (Kwon *et al.*, 2001). A comprehensive review of the thermal imaging technique in the heat transfer research can be found elsewhere (Valvano, 1992).

An elliptic inverse problem is generally easier to solve than a parabolic one because instability caused by time marching is absent. However, errors in the temperature measurement can be amplified drastically with increasing distance between the locations of measurement and estimation, and with increasing heat exchange rate on the measured boundary. Therefore, a smoothing technique is essential in achieving the acceptable inverse solution. In this work, the unknown temperature of interest is approximated with a B-spline surface. The order and number of the base functions can control the smoothness. This approach can provide an accurate solution with admissible smoothness that is statistically consistent with the measurement data. This study presents a derivation of gradient-based formulation for a three-dimensional steady IHCP using B-spline function specification. The proposed method is tested for a three-dimensional slab with uniform thickness and constant thermal conductivity.

Problem statement and formulation

Direct problem

Consider an arbitrary three-dimensional domain Ω shown in Figure 1. The governing equation for steady-state heat conduction with temperature-independent thermal conductivity and without internal heat generation is written as follows.

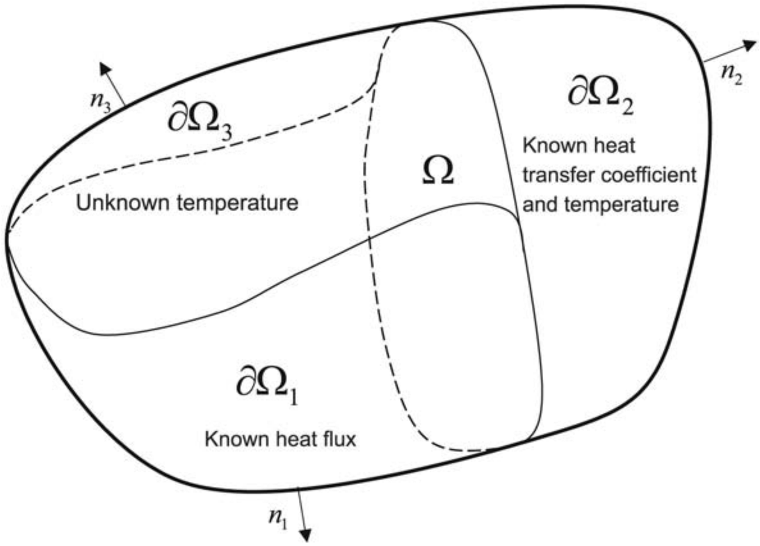

Figure 1.
Arbitrary three-dimensional domain considered in this study

$$\nabla(k \cdot \nabla T) = 0, \quad \text{in } \Omega \tag{1a}$$

The boundary conditions on each boundary segment $\partial\Omega_i$ (i=1, 2, 3) are

$$\frac{\partial T}{\partial n_1} = 0, \quad \text{on } \partial\Omega_1 \tag{1b}$$

$$\frac{\partial T}{\partial n_2} = \frac{h}{k}[T(\mathbf{r}) - T_\infty], \quad \text{on } \partial\Omega_2 \tag{1c}$$

$$T(\mathbf{r}) = f(\mathbf{r}), \quad \text{on } \partial\Omega_3 \tag{1d}$$

Here, k is the thermal conductivity, T_∞ is the ambient temperature and h is the heat transfer coefficient. The heat transfer coefficient is well known in many engineering situations as a result of accumulated research for the last several decades (Rohsenow *et al.*, 1998). Besides, the temperature on $\partial\Omega_3$, $f(\mathbf{r})$ should be determined through the inverse method.

Sensitivity problem

When the unknown f is varied by a small amount Δf, T undergoes a perturbation $T + v$. Then, replacing T in the original governing equations by $T + v$, f by $f + \Delta f$, perturbed equations are obtained. Subtracting the original equations from the perturbed gives the following sensitivity equations.

$$\nabla(k \cdot \nabla v) = 0, \quad \text{in } \Omega \tag{2a}$$

$$\frac{\partial v}{\partial n_1} = 0, \quad \text{on } \partial\Omega_1 \tag{2b}$$

$$\frac{\partial T}{\partial n_2} = \frac{h}{k} v(\mathbf{r}), \quad \text{on } \partial\Omega_2 \tag{2c}$$

$$v(\mathbf{r}) = \Delta f(\mathbf{r}), \quad \text{on } \partial\Omega_3 \tag{2d}$$

Inverse problem and measurement

The inverse problem aims to achieve a solution that renders consistency between the measured and computed temperatures. Such consistency can be measured by the following residual functional.

$$J(f) = \int_{\partial\Omega_2} [T(\mathbf{r};f) - Y(\mathbf{r})]^2 \, \mathrm{d}S \tag{3}$$

where $T(\mathbf{r};f)$ is temperature dependent on the unknown temperature $f(\mathbf{r})$ and $Y(\mathbf{r})$ is the measured temperature. The admissible target value of the residual functional is

$$J \approx \int_{\partial\Omega_2} \sigma^2 \, \mathrm{d}S \tag{4}$$

where σ is the standard deviation of the measured temperature.

Adjoint problem

In order to achieve a solution satisfying the above criterion, the gradient of J must be evaluated. The gradient can be obtained by utilizing the solution of the adjoint problem. Let us introduce Lagrange multiplier λ to derive the adjoint problem. To obtain the Lagrangian function, equations (1(a)) and (3) are treated as an objective and a constraint, respectively. Multiply equation (1(a)) by λ and integrate the result over the entire domain. Adding this expression to the original residual functional gives

$$J(f) = \int_{\partial\Omega_2} [T(\mathbf{r};f) - Y(\mathbf{r})]^2 \, \mathrm{d}S + \int_{\Omega} \lambda \nabla (k \cdot \nabla T) \, \mathrm{d}V \tag{5}$$

Considering the increment $\Delta J \equiv J(f + \Delta f) - J(f)$ and neglecting the terms of the second order, we obtain the following variation of the residual J.

$$\Delta J = \int_{\partial\Omega_2} 2[T(\mathbf{r};f) - Y(\mathbf{r})] v(\mathbf{r}) \, \mathrm{d}S + \int_{\Omega} \lambda \nabla (k \cdot \nabla v) \, \mathrm{d}V \tag{6}$$

Integrating the second integral

$$\int_{\Omega} \lambda \nabla (k \cdot \nabla v)\, \mathrm{d}V$$

by parts with the Green's formula gives

$$\int_{\partial\Omega_{1,2,3}} \lambda (k \cdot \nabla v)\, \mathrm{d}V - \int_{\Omega} \nabla \lambda (k \cdot \nabla v)\, \mathrm{d}V$$

Then, integrating this by parts again, we have

$$\Delta J = \int_{\partial\Omega_2} 2[T(\mathbf{r};f) - Y(\mathbf{r})] v(\mathbf{r})\, \mathrm{d}S + \int_{\partial\Omega_{1,2,3}} \lambda (k \cdot \nabla v)\, \mathrm{d}V$$
$$- \int_{\partial\Omega_{1,2,3}} v (k \cdot \nabla \lambda)\, \mathrm{d}V + \int_{\Omega} v \nabla (k \cdot \nabla \lambda)\, \mathrm{d}V \tag{7}$$

Utilizing the arbitrariness of a state variable v and the boundary conditions of the sensitivity problem, equations (2b) and (2c), the following adjoint problem is obtained.

$$\nabla (k \cdot \nabla \lambda) = 0, \quad \text{in } \Omega \tag{8a}$$

$$\frac{\partial \lambda}{\partial n_1} = 0, \quad \text{on } \partial\Omega_1 \tag{8b}$$

$$\frac{\partial \lambda}{\partial n_2} = \frac{h}{k}[\lambda(\mathbf{r}) - T_{\infty}] - 2[T(\mathbf{r};f) - Y(\mathbf{r})], \quad \text{on } \partial\Omega_2 \tag{8c}$$

$$\lambda(\mathbf{r}) = 0, \quad \text{on } \partial\Omega_3 \tag{8d}$$

Then, using equations (8) and (2d), equation (7) becomes

$$\Delta J = - \int_{\partial\Omega_3} \frac{\partial \lambda}{\partial n_3} \Delta f\, \mathrm{d}S \tag{9}$$

The following relation holds by definition in the square-integrable function space (L-2 space) (Alifanov, 1994).

$$\Delta J = \int_{\partial\Omega_3} J'(\mathbf{r}) \Delta f\, \mathrm{d}S \tag{10}$$

Comparing the above two equations gives the following gradient equation.

$$J'(\mathbf{r}) = -\frac{\partial\lambda}{\partial n_3} \tag{11}$$

Here, J' is the gradient of the functional J.

B-spline function specification and gradient

The surface temperature is represented by a B-spline surface as follows (Piegl and Tiller, 1997).

$$f(\mathbf{r}) = \sum_{k=1}^{N} \tilde{f}_k \varphi_k(\mathbf{r}), \quad \text{for } k = 1, \ldots, N \tag{12}$$

where $\varphi_k(\mathbf{r})$ is the k-th base function, which is given by

$$\varphi_k(\mathbf{r}) = \vartheta_p(s)\vartheta_q(t), \quad \text{for } p = 1, \ldots, P, \quad q = 1, \ldots, Q, \\ k = p + (q-1)P \tag{13}$$

where $\vartheta_p(s)$ and $\vartheta_q(t)$ are the uniform periodic B-spline blending functions with order O, which can be evaluated using the de Boor's (1978) recursive formula. s- and t- directions are perpendicular to each other and tangent to the surface at a given point $\mathbf{r}$ on the boundary $\partial\Omega_3$. P and Q are numbers of control points in s- and t-directions, respectively, and N is the number of the base functions ($N = P \times Q$). The coefficients $\tilde{f}_k$ are the unknowns of the inverse problem. The variation of $f(\mathbf{r})$ and $\Delta f(\mathbf{r})$ is given by

$$\Delta f(\mathbf{r}) = \sum_{k=1}^{N} \Delta\tilde{f}_k \varphi_k(\mathbf{r}) \tag{14}$$

Substituting the above expression in equation (10), ΔJ takes the form

$$\Delta J = \int_{\partial\Omega_3} J'(\mathbf{r}) \sum_{k=1}^{N} \Delta\tilde{f}_k \varphi_k(\mathbf{r})\,\mathrm{d}S \tag{15}$$

The following expression for ΔJ is evident.

$$\Delta J = \sum_{k=1}^{N} \frac{\partial J}{\partial \tilde{f}_k} \Delta\tilde{f}_k \tag{16}$$

Comparing equations (15) and (16) gives the following equation for gradient.

$$\frac{\partial J}{\partial \tilde{f}_k} = \int_{\partial\Omega_3} J'(\mathbf{r})\varphi_k(\mathbf{r})\,\mathrm{d}S \tag{17}$$

Solution scheme

Numerical discretization

Various numerical schemes can be utilized to solve the direct, sensitivity, and adjoint problems. For efficiency and accuracy, the BEM is employed in this study. BEM is suitable for the proposed method since solutions of direct, sensitivity and adjoint problems are not required for the interior region. The governing equations are linear and the fundamental solution is available. Furthermore, the gradient $J'(\mathbf{r})$ (equation (11)) is directly obtained from BEM without having to perform further calculation. Brebbia *et al.* (1984) has described the way of implementing BEM for three-dimensional potential problems. It is noted that the boundary region in this study is discretized with bilinear quadrilateral elements. Computational and implementational aspects of BEM for the steady IHCP are well described in a previous study (Mera *et al.*, 2001).

Residual optimization

The conjugate gradient method (CGM) for the iterative inverse analysis has been addressed in previous works (Alifanov, 1994; Jarny *et al.*, 1991). The following is a solution algorithm based on CGM for the IHCP in this study.

(1) Give an initial guess.

(2) Update temperature field by solving equations (1).

(3) If the statistical consistency given by equation (4) is satisfied, terminate the procedure.

(4) Using the solution of adjoint problem given by equations (8), evaluate the gradient by equation (17).

(5) Evaluate the conjugate coefficient by the following expression (Jarny *et al.*, 1991).

$$\gamma = \mathbf{g} \cdot (\mathbf{g} - \mathbf{g}^p)/\mathbf{g}^p \cdot \mathbf{g}^p \tag{18}$$

where $\mathbf{g} \equiv \{g_1, \ldots, g_k, \ldots, g_N\}$ and p denotes the previous iteration. Here,

$$g_k = \frac{\partial J}{\partial \tilde{f}_k}.$$

(6) Update the conjugate direction by the following expression.

$$\mathbf{S} = \mathbf{S}^p + \gamma \mathbf{g} \tag{19}$$

where $\mathbf{S}$ is initially a null vector.

(7) Evaluate Δf using equation (14).

(8) Solve the sensitivity problem and obtain $v(\mathbf{r};\Delta f)$.

(9) By using the solution of the sensitivity problem, obtain the step length by the following expression (Jarny *et al.*, 1991).

$$\beta = \frac{\int_{\partial\Omega_2} [T(\mathbf{r};f) - Y(\mathbf{r})] v(\mathbf{r};\Delta f)\, \mathrm{d}S}{\int_{\partial\Omega_2} v(\mathbf{r};\Delta f)^2\, \mathrm{d}S} \tag{20}$$

(10) Update the coefficients $\tilde{f}_k$ by

$$\mathbf{f} = \mathbf{f}^p + \beta \mathbf{S} \tag{21}$$

where $\mathbf{f} \equiv \{\tilde{f}_1, \ldots, f_k \ldots, f_N\}$

(11) Update the unknown surface temperature by equation (12), then go to 2.

Results and discussion

Test specification

In order to verify the proposed method, a three-dimensional slab shown in Figure 2 is considered. The dimension of the slab is given by $L = W = 1\,\mathrm{m}$ and $H = 0.1\,\mathrm{m}$ where L, W and H are length, width and height, respectively, as shown in Figure 2. A Cartesian coordinate system is used and the coordinate variables x, y and z are as denoted in the figure. If not specified, the heat transfer coefficient of the bottom surface is given as $10\,\mathrm{W/m^{2\circ}C}$ ($\mathrm{Bi}_H \equiv hH/k = 1$). Thermal conductivity k is $1\,\mathrm{W/m^\circ C}$ over the entire domain. The lateral sides correspond to $\partial\Omega_1$ and are insulated.

The test cases are run with a boundary mesh system, which is structured, orthogonal, and comprised of equi-spaced nodes. A total of 1,602 boundary nodes are used for the estimation: $M_x = M_y = 21$ and $M_z = 11$, where M_x, M_y and M_z are number of nodes in x-, y- and z- directions, respectively. All the boundary elements are bilinear-quadrilateral elements.

Tests have been performed using artificially generated measurement data with and without errors. The measurement data are assumed to be available at discrete points (nodal points on $\partial\Omega_2$). The errors are artificially embedded in the exact data using the following equation.

$$Y_i = Y_{e,i} + \sigma r_i, \quad i = 1, \cdots, M_x \times M_y \tag{22}$$

where the subscript i denotes i-th nodal point on the bottom surface, r_i is a normally distributed random variable with zero mean and unit standard deviation within 99 per cent confidence interval (i.e. $-2.576 < r_i < 2.576$), and σ is the standard deviation. The random variable is generated by *random_normal* an IMSL® C function (Visual Numerics, 1997). The exact temperature $Y_{e,i}$ is obtained from the numerical solution of the corresponding direct problem.

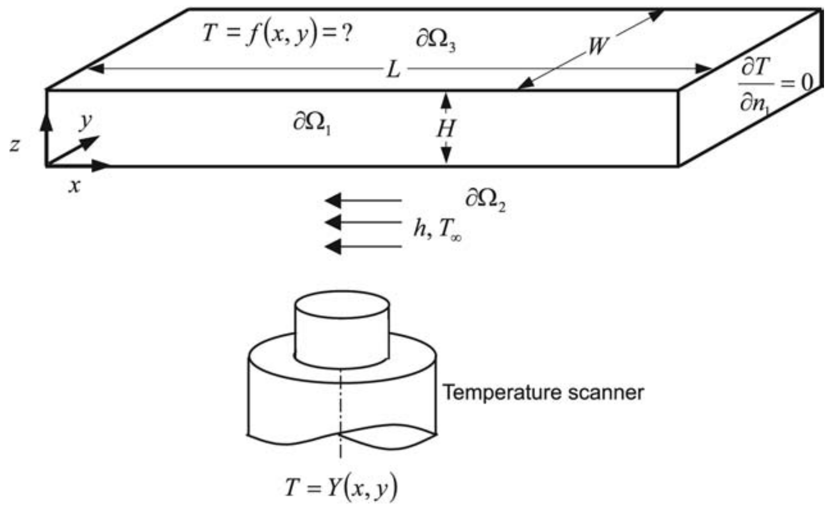

Figure 2.
A three-dimensional slab investigated in the test case

The order of the base function O and the number of the base functions N are parameters that control the smoothness and resolution. The inverse estimation of the surface temperature is carried out for several different Os and Ns. The test cases include estimations of temperature distribution with sinusoidal and rectangular forms. The sinusoidal form can be considered as an example form of smooth distribution (see Figure 3). The exact form of the unknown temperature $f_e(\mathbf{r}) = f_e(x, y)$ is given by

$$f_e(x,y) = f_o \times \left[0.5\ \sin\left(2\pi\frac{x}{L} - 0.5\pi\right) + 0.5\right] \times \left[0.5\ \sin\left(2\pi\frac{y}{W} - 0.5\pi\right) + 0.5\right] \quad (23)$$

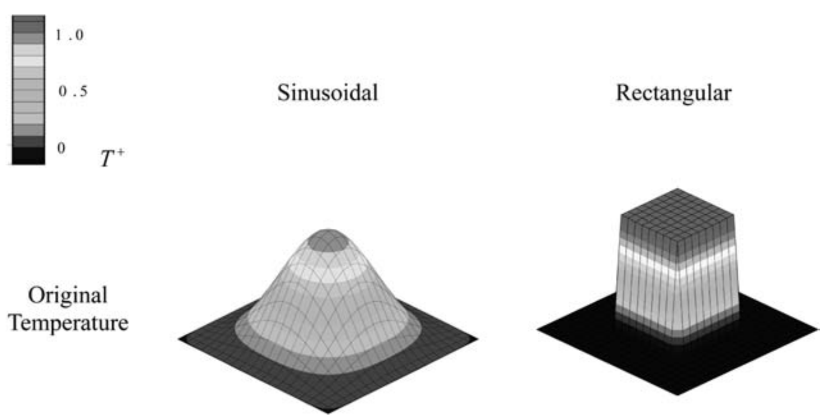

Figure 3.
Original temperature profiles to be estimated on the top surface of the slab

The rectangular temperature distribution (see Figure 2) is given as

$$f(x,y) = f_o \times \left[u\left(x - \frac{3L}{10}\right) - u\left(x - \frac{7L}{10}\right)\right] \times \left[u\left(y - \frac{3W}{10}\right) - u\left(y - \frac{7W}{10}\right)\right] \tag{24}$$

where u is the step function and f_o is a positive nominal temperature of the top surface. For the presentation of computational results, the following dimensionless variables are introduced.

$$T^+ \equiv \frac{T - T_\infty}{f_o - T_\infty}, \qquad Y^+ \equiv \frac{Y - T_\infty}{f_o - T_\infty}, \qquad \sigma^+ \equiv \frac{\sigma}{f_o} \tag{25}$$

Tests are performed for different standard deviations ($\sigma^+ = 0$, 0.01, 0.05 and 0.1), orders of the base function ($N = 3$, 4 and 5), Biot numbers ($\mathrm{Bi}_H = 1$, 5 and 25) and numbers of the base functions ($N = 8 \times 8$, 10×10, 12×12, 14×14, 16×16 and 18×18).

Furthermore, the accuracy of inverse estimation is measured by the following residual expression.

$$D = \sqrt{\frac{\int_{\partial\Omega_3} [f(\mathbf{r}) - f_e(\mathbf{r})]^2 \, \mathrm{d}S}{\int_{\partial\Omega_3} \mathrm{d}S}} \tag{26}$$

Noisy measurement data and estimated temperature

Figure 4 shows the result estimated using exact and noisy measurement data for the rectangular form. For the exact measurement case, the estimated temperature simulates the original temperature with certain degree of bias. Such bias is called deterministic bias, which is a nature of an inverse estimator and can be assessed by D (Beck *et al.*, 1985). The deterministic bias of the proposed method is investigated quantitatively later in this paper. The result for $\sigma^+ = 0.01$ is obtained with a reduced N. This result shows a shape similar to the result for $\sigma^+ = 0$. However, D is increased due to increased noise in the measurement and the change of N as shown in the figure. For $\sigma^+ = 0.1$, despite the fact that the measurement data is severely impaired, a smooth temperature distribution is attained by further reduction of N ($N_R = 8 \times 8$). The random errors in the measurement data incur noticeable resolution loss and increased bias as in this case.

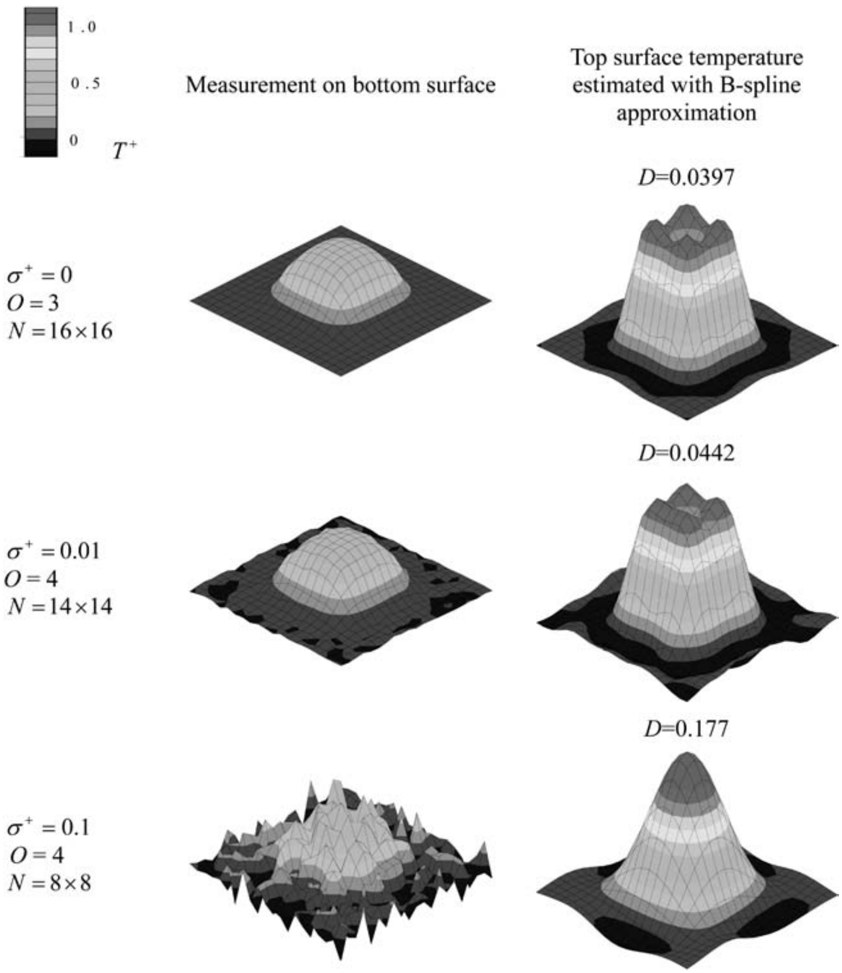

Figure 4.
Inverse estimation of surface temperature with B-spline function specification using exact and noisy measurement data

Comparison between the results with and without parameterization

In order to verify the effectiveness of the B-spline function specification, the result is compared to the method without it. As can be seen in Figure 5, the results by both methods for $\sigma^+ = 0$ and $\sigma^+ = 0.01$ are comparable. Although the function specification is not used, some smoothing is still present by virtue of the viscous nature of the conjugate gradient method (Alifanov, 1994). The result for $\sigma^+ = 0$ shows the deterministic biases of the estimation with and without the parameterization. It is observed that the function specification induces further deterministic error. D is considerably increased with the function specification (from 0.298 to 0.374). Thus, an analysis for deterministic

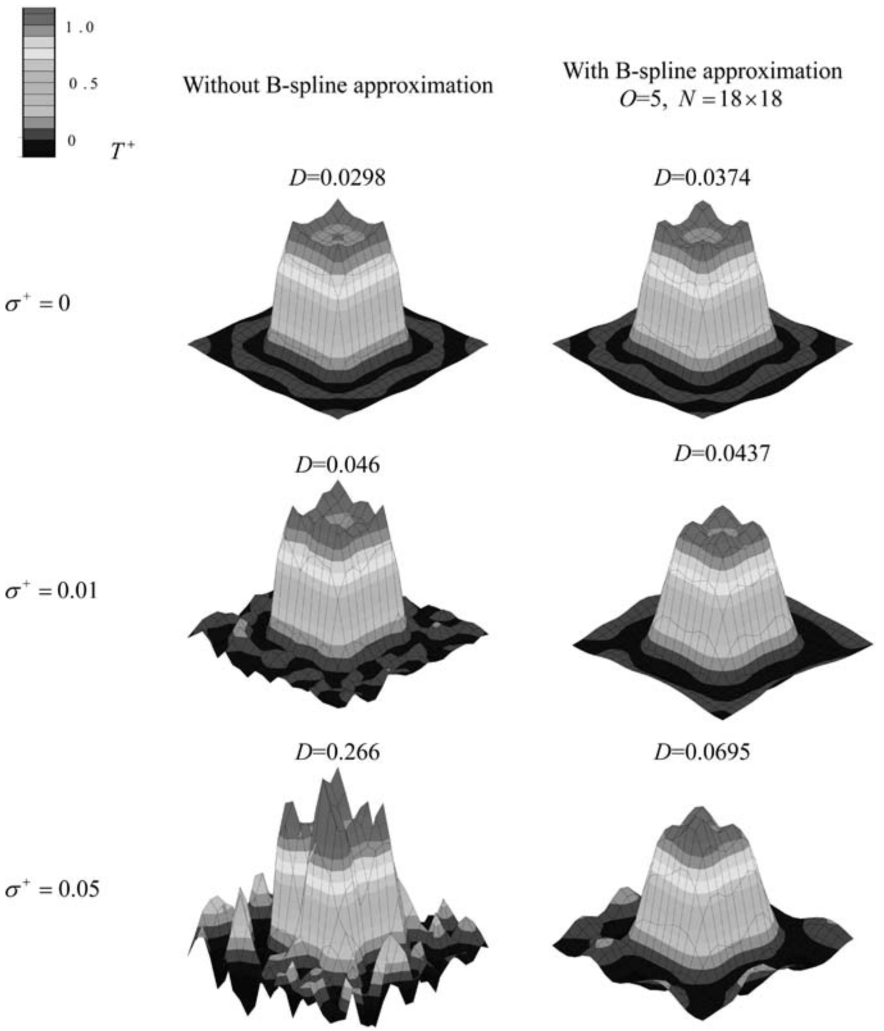

Figure 5.
Inverse estimation of rectangular surface temperature with and without B-spline approximation using noisy measurement data with varying error level

bias is important for characterizing the proposed method. However, for $\sigma^+ = 0.01$, smaller D is obtained with the function specification. Moreover, for $\sigma^+ = 0.05$, as shown in the figure, the result without the function specification is unacceptably fluctuating and D is increased significantly ($D = 0.266$). Using the function specification, a well-stabilized result with smaller D ($D = 0.0695$) is obtained.

When a parameterization technique such as the B-spline function specification is not used, the conventional regularization method can be incorporated for similar inverse problems (Jarny *et al.*, 1991). The determination of the regularization parameter is not an easy task, which usually requires feedback. The selection of N and O is more intuitive than the determination of the regularization parameter and the order of the regularization. However, the B-spline function specification method cannot control the smoothness of the solution in a continuous manner as can the regularization method. When fine and continuous control of smoothness is essential, the regularization method can be combined with the B-spline function specification method.

Deterministic bias

Although the exact measurement data are utilized for the estimation, the result shows certain deviation from the exact temperature because of the deterministic bias. Such bias is investigated for several different Os and Ns. The results for the sinusoidal form are shown in Figure 6. These results are almost indistinguishable from its original form. D is also very small for all the results. The smooth shape is generally easier to reconstruct than shapes with abrupt changes. The results for rectangular form show noticeable deviation

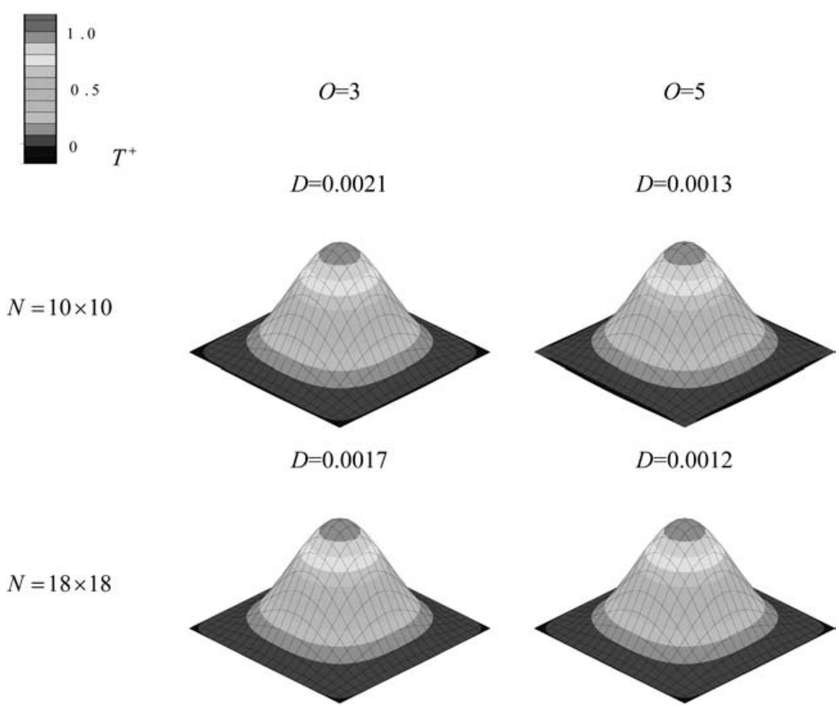

Figure 6.
Inverse estimation of sinusoidal surface temperature with B-spline approximation for different numbers of base functions and order using exact measurement data

from its original form as can be seen in Figure 7. For $O = 3$ and $N = 10 \times 10$, sharp overshoots are observed at the four corners on the rising edges because the region covered by one base function is excessively wide to represent with such a low order base function. Increasing N can reduce the region covered by one base function. As a result, such a large overshoot is alleviated by increasing N from 10×10 to 14×14 as can be seen in Figure 7. However, for $O = 5$, when N is increased from 10×10 to 14×14, decrease in D is considerably smaller than for $O = 3$. Figure 8 shows change of D with increasing N for $O = 3$ and 5. The deterministic errors for $O = 3$ and 5 become closer to each other as N increases.

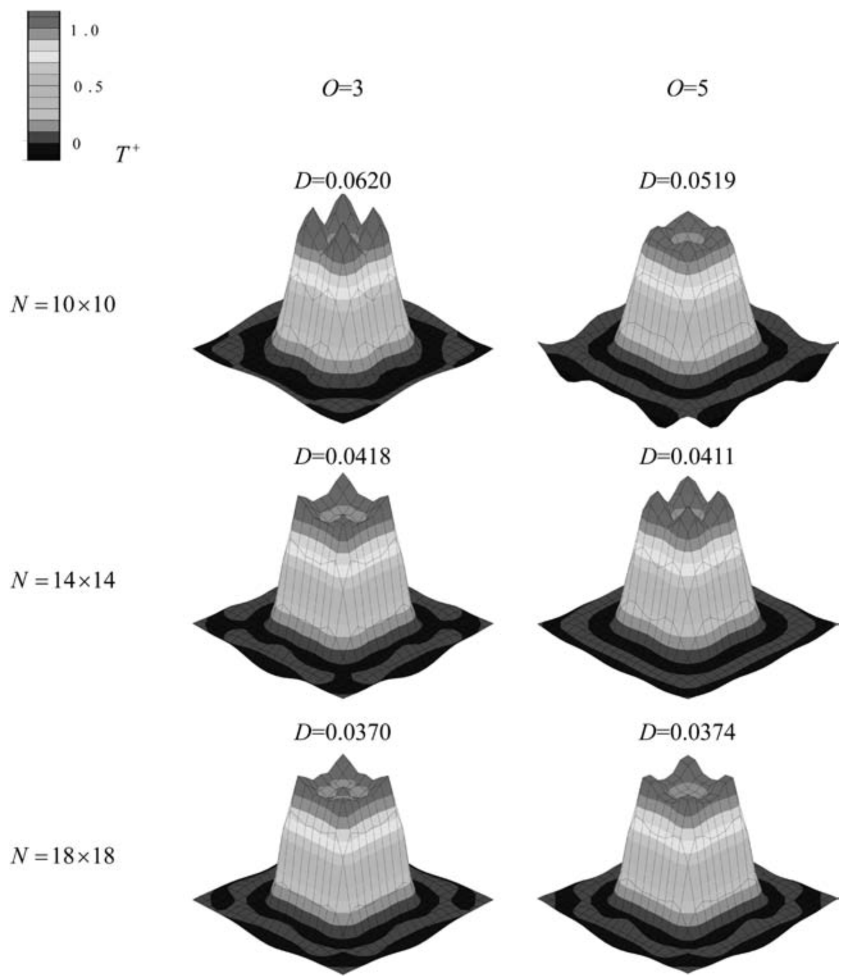

Figure 7.
Inverse estimation of rectangular surface temperature with B-spline approximation for different numbers of base functions and order using exact measurement data

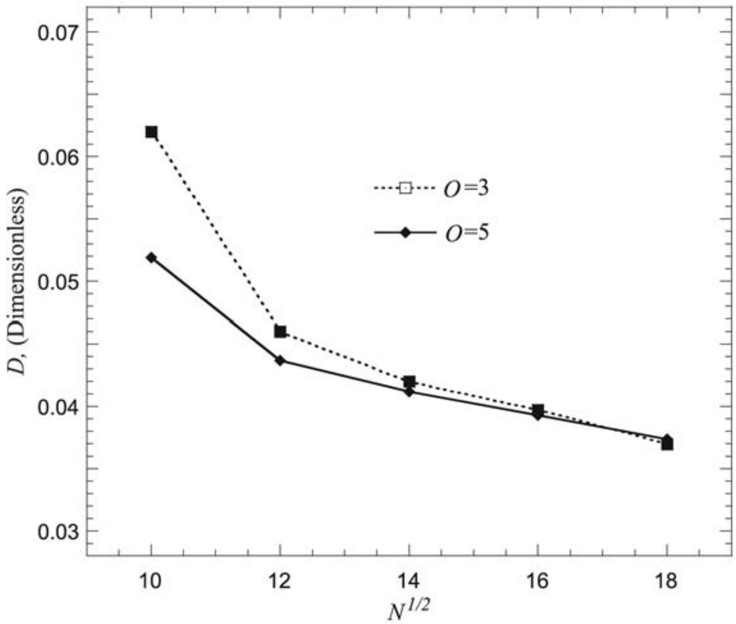

Figure 8.
Deterministic bias D versus $N^{1/2}$

Basically using a higher order base function is advantageous in representing surfaces with mild curvature as smaller D is obtained for larger O as in the sinusoidal case (see Figure 6). Sharp edges are better depicted with $O = 3$ than with $O = 5$ as can be seen in Figure 7. This smoothing issue has been extensively studied for one-dimensional problems in previous studies (Alifanov *et al.*, 1987; Flach and Özişik, 1987). These works for one-dimensional problems can provide useful information for multi-dimensional cases. Whenever possible, N and O should be determined utilizing *a priori* information about the unknown temperature distribution.

Effect of Biot number

To examine the influence of change in thickness H, and the heat transfer coefficient h, inverse estimations are conducted for different Biot numbers. Figure 9 shows the results for $Bi_H = 1$, 5 and 25. With errorless measurements, the effect of Biot number is negligible. However, with impaired measurement data ($\sigma^+ = 0.01$), the overall error of the solution is augmented with the increase of Bi_H as can be seen in the figure.

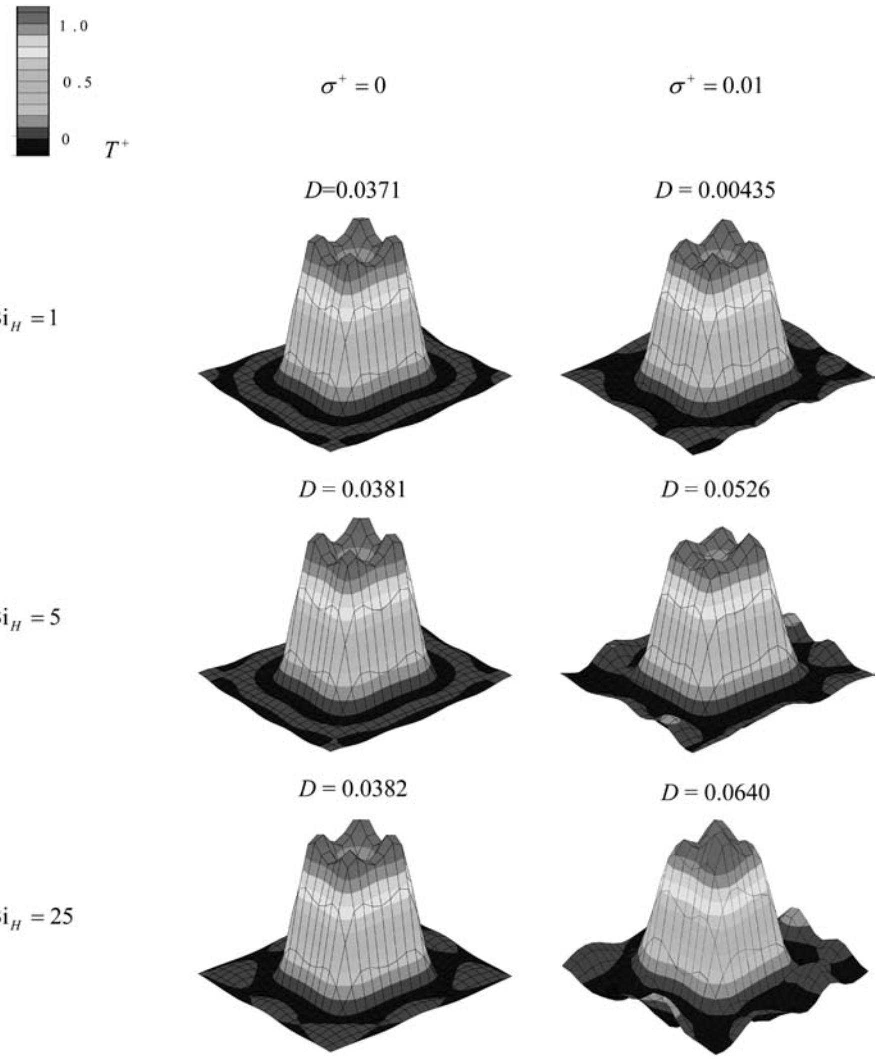

Figure 9.
Inverse estimation of rectangular surface temperature with B-spline approximation ($O = 4$ and $N = 18 \times 18$) using exact and noisy measurement data ($\sigma^+ = 0.01$) for different Biot numbers

Conclusions

The inverse estimation of steady-state surface temperature from measured temperature is investigated for a three-dimensional body. A gradient-based method with B-spline function specification for the steady IHCP is described. The validity and characteristics of the proposed method are examined with computational results for a three-dimensional slab. It is shown that the error propagation during the inverse estimation can be effectively reduced by selecting the proper number and order of B-spline base functions. Furthermore, B-spline representation of the inverse solution provides reasonable accuracy by localizing the abrupt variation in the solution.

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